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BOSTON PUBLIC SCHOOLS

ARITHMETIC

DETERMINING THE ACHIEVEMENT OF PUPILS IN THE ADDITION OF FRACTIONS

BULLETIN NO. VII. OF THE DEPARTMENT OF EDUCATIONAL INVESTIGATION AND MEASUREMENT

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IN SCHOOL COMMITTEE, BOSTON, February 7, 1916.

Ordered, That four thousand (4,000) copies of a bulletin relative to a test in the addition of fractions, to be prepared by the Department of Educational Investigation and Measurement, be printed as a school document.

Attest:

THORNTON D. APOLLONIO,
Secretary.

INTRODUCTION.

During the past few years educational measurement has been applied to arithmetic more than to any other subject, largely because of the ease with which papers can be scored. This application of educational measurement has been largely limited to work with integers. In Boston, the Courtis Standard Tests, Series A or B, have been given since 1912. Until December, 1915, no other phases of arithmetic work were subjected to scientific measurement.

Inasmuch as a system for giving the Courtis tests has been worked out, and also because definite improvement in the use of whole numbers is being secured, the time seemed appropriate for extending educational measurement to other phases of arithmetic.

As a result, a group of teachers and masters was selected in December, 1914, to constitute a Committee on Standards in Arithmetic, for the purpose of making a study of the problems involved in the addition, subtraction, multiplication and division of fractions. The committee consists of the following named members:

Clarence H. Jones, Sub-master, Martin District, *Chairman.*
Gertrude E. Bigelow, Master, Hancock District.
Alton C. Churbuck, Sub-master, Quincy District.
John J. Cummings, Sub-master, Oliver Wendell Holmes District.
Arthur L. Gould, Master, Dearborn District.
Ellen M. Greany, Assistant, Hugh O'Brien District.
Annie R. Mohan, Master's Assistant, Emerson District.
William L. Vosburgh, Head of the Department of Mathematics, Normal School.

The results of the work of the committee, together with the results of an attempt of the department to

ascertain what the ability of Boston pupils to add fractions is, are reported in this bulletin. Mr. Arthur W. Kallom kept in close touch with the work of the committee, prepared the tests to be given by the department, helped to train the examiners to give them, supervised the tabulation of the results, and prepared the manuscript for publication.

FRANK W. BALLOU,
Director.

DETERMINING THE ACHIEVEMENT OF PUPILS IN THE ADDITION OF FRACTIONS.

I. The Work of the Committee on Standards.

As soon as the "Committee on Standards in Arithmetic" began work, it found that the problem of fractions was a complicated one. The committee devoted several months to the study of the problem, most of the time being consumed in the study of addition of fractions. In its study the committee distinguished fourteen distinct types. The classification is based on two factors, the common denominator and the form of the answer.

Analysis of Types of Fractions.

All fractions may be divided into two classes, similar and dissimilar. Similar fractions are defined as fractions having the same denominator. Dissimilar fractions are those which have different denominators. Each one of these classes may be further subdivided according to whether the result obtained from adding is non-reducible or reducible. If the result is non-reducible, the answer is in an acceptable form after the numerators of the similar fractions have been added. If the result is reducible, it may be changed to a mixed number, to lowest terms, or to both, before the answer can be said to be in an acceptable form. A discussion of the fourteen types is here given with the hope that it may be of value to teachers in instructing children how to add fractions.

A. Similar Fractions.

Four types are all that are possible when fractions are similar.

Type 1.— After adding, the result is non-reducible.

For example: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$.

Type 2.— After adding, the result is reducible either to an integer or to a mixed number.

For example: $\frac{4}{7} + \frac{4}{7} = \frac{8}{7} = 1\frac{1}{7}$.

Type 3.— After adding, the result is reducible to the lowest terms.

For example: $\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$.

Type 4.— After adding, the result is reducible both to the lowest terms and to a mixed number.

For example: $\frac{5}{6} + \frac{5}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$.

B. Dissimilar Fractions.

Under the heading of dissimilar fractions there are three divisions depending on the common denominator.

(*a.*) When the least common denominator is the denominator of one of the fractions.

(*b.*) When the least common denominator is the product of the denominators.

(*c.*) When the least common denominator is found by factoring.

These will be considered in order.

(*a.*) When the least common denominator is the denominator of one of the fractions, there are four types, one non-reducible and three reducible, similar to the preceding four types.

Type 5.— The denominator of one of the fractions is the least common denominator. After adding, the result is non-reducible.

For example: $\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$.

Type 6.— The denominator of one of the fractions is the least common denominator. After adding, the result is reducible to a mixed number.

For example: $\frac{2}{5} + \frac{7}{10} = \frac{11}{10} = 1\frac{1}{10}$.

Type 7.— The denominator of one of the fractions is the least common denominator. After adding, the result is reducible to lowest terms.

For example: $\frac{1}{2}+\frac{3}{10}=\frac{8}{10}=\frac{4}{5}$.

Type 8.— The denominator of one of the fractions is the least common denominator. After adding, the result is reducible to lowest terms and to a mixed number.

For example: $\frac{3}{4}+\frac{5}{12}=\frac{14}{12}=\frac{7}{6}=1\frac{1}{6}$.

(*b*.) When the least common denominator is the product of the denominators, there are only two types, one non-reducible and the other reducible to a mixed number. After reducing fractions of these two types to similar fractions and adding the numerators, the result is always a fraction in which the numerator and denominator are prime to each other. This makes it impossible to obtain a result which can be reduced to lowest terms.

Type 9.— The denominators are prime to each other, hence the common denominator must be the product of the denominators. After adding, the result is non-reducible.

For example: $\frac{2}{5}+\frac{3}{8}=\frac{31}{40}$.

Type 10.— The denominators are prime to each other, hence the common denominator must be the product of the denominators. After adding, the result is reducible to a mixed number.

For example: $\frac{7}{9}+\frac{1}{4}=\frac{37}{36}=1\frac{1}{36}$.

(*c*.) When the least common denominator is found by factoring, there are four types, one non-reducible and three reducible.

Type 11.— The least common denominator is found by factoring. After adding, the result is non-reducible.

For example: $\frac{1}{6}+\frac{2}{9}=\frac{7}{18}$.

Type 12.— The least common denominator is found by factoring. After adding, the result is reducible to a mixed number.

For example: $\frac{1}{4} + \frac{5}{6} = \frac{13}{12} = 1\frac{1}{12}$.

Type 13.— The least common denominator is found by factoring. After adding, the result is reducible to lowest terms.

For example: $\frac{1}{6} + \frac{2}{15} = \frac{9}{30} = \frac{3}{10}$.

Type 14.— The least common denominator is found by factoring. After adding, the result is reducible to lowest terms and to a mixed number.

For example: $\frac{1}{6} + \frac{9}{10} = \frac{32}{30} = \frac{16}{15} = 1\frac{1}{15}$.

Summary.

The following is a summary of the fourteen types as they were classified by the committee:

A.— Similar Fractions.

Non-reducible.

Type 1.— Answer in final form.

Example: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$.

Reducible.

Type 2.— To integers or mixed numbers.

Example: $\frac{4}{7} + \frac{4}{7} = \frac{8}{7} = 1\frac{1}{7}$.

Type 3.— To lowest terms.

Example: $\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$.

Type 4.— To lowest terms and mixed numbers.

Example: $\frac{5}{6} + \frac{5}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$.

B.— Dissimilar Fractions.

(*a*) Least common denominator the denominator of one of the fractions.

Non-reducible.

Type 5.— Answer in final form.

Example: $\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$.

Reducible.

Type 6.— To mixed numbers.

Example: $\frac{2}{5} + \frac{7}{10} = \frac{11}{10} = 1\frac{1}{10}$.

Type 7.— To lowest terms.

Example: $\frac{1}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$.

Type 8.— To lowest terms and mixed numbers.

Example: $\frac{3}{4} + \frac{5}{12} = \frac{14}{12} = \frac{7}{6} = 1\frac{1}{6}$.

(*b*) Least common denominator the product of the denominators.

Non-reducible.

Type 9.— Answer in final form.

Example: $\frac{2}{5} + \frac{3}{8} = \frac{31}{40}$.

Reducible.

Type 10.— To mixed numbers.

Example: $\frac{2}{3} + \frac{3}{4} = \frac{17}{12} = 1\frac{5}{12}$.

(*c*) Least common denominator found by factoring.

Non-reducible.

Type 11.— Answer in final form.

Example: $\frac{1}{6} + \frac{2}{9} = \frac{7}{18}$.

Reducible.

Type 12.— To mixed numbers.

Example: $\frac{1}{4} + \frac{5}{6} = \frac{13}{12} = 1\frac{1}{12}$.

Type 13.— To lowest terms.

Example: $\frac{1}{6} + \frac{2}{15} = \frac{9}{30} = \frac{3}{10}.$

Type 14.— To lowest terms and mixed numbers.

Example: $\frac{5}{10} + \frac{9}{14} = \frac{80}{70} = \frac{8}{7} = 1\frac{1}{7}.$

Experimental Work of the Committee.

The committee prepared a comprehensive list of examples under each type and each member was asked to furnish this list to a fifth grade teacher and to secure her coöperation in this work.

Addition of Fractions. Form 1.

Score............... Time............... Name.......................................

(3) $\frac{3}{16}$ $\frac{7}{16}$	(4) $\frac{5}{6}$ $\frac{5}{6}$	(5) $\frac{2}{3}$ $\frac{4}{15}$	(6) $\frac{3}{4}$ $\frac{7}{8}$
(7) $\frac{1}{2}$ $\frac{3}{10}$	(8) $\frac{6}{7}$ $\frac{9}{14}$	(9) $\frac{2}{9}$ $\frac{7}{10}$	(10) $\frac{1}{4}$ $\frac{8}{9}$
(11) $\frac{3}{8}$ $\frac{7}{12}$	(12) $\frac{7}{9}$ $\frac{8}{15}$	(13) $\frac{1}{6}$ $\frac{7}{10}$	(14) $\frac{5}{6}$ $\frac{3}{14}$

In order that each teacher might have a means of determining in which type her children were weakest, a form of twelve examples was prepared like that illustrated above. One example was selected from each of Types 3 to 14, inclusive. That is, the first example in the illustration belongs to Type 3, the second to Type 4, and so on, as shown by the numbers. Types 1 and 2 were omitted because of their relative simplicity.

These examples were printed on a sheet 8 inches by $5\frac{1}{2}$ inches. Space was left at the right and below each example in which the child could do any work which he wished to do. Ten different forms were printed and enough of them were furnished each teacher to supply her class. These forms were used by the teachers as practice material.

Form 1 was given to the class by the teacher and the results obtained showed the strength and weakness of each pupil. If the children failed on the example representing Type 5, the teacher had a number of examples in the list prepared by the committee which would give them material for practice on this particular type. Other children who failed on Type 10 likewise had a large number of examples for their particular need. After a few days of practice in helping children where they needed most help, Form 2 was given and the progress of individuals could be noted. The children were allowed all the time they needed to complete the examples. A record of the time was kept by the teacher. After completing the form, the examples were corrected, and the score in examples right and the time required was recorded in the proper space. A record of each individual was kept during the entire experiment, showing the accuracy with which he performed the work and the time required to do it.

In June the results were sent to the office of the department for analysis. In general the accuracy increased and the time necessary to do the examples shortened as the children worked each successive form. In addition to this, the data gathered from this material showed:

(*a*.) That testing of this kind would be likely to produce results which would show the ability of children to add fractions.

(*b*.) That children might be able to do non-reducible types but would have trouble in doing types where reduction was necessary.

(*c*.) That the time required to do the twelve examples

on each form varied greatly. The shortest time recorded by any individual was two minutes, the longest forty-one minutes.

II. THE WORK OF THE DEPARTMENT.

Planning the Testing.

a. Selection of Schools.

In selecting the schools in which the tests in fractions should be given, it was necessary to take into consideration two conditions:

(*a.*) That at least 1,000 children should be tested in each grade.

(*b.*) That these 1,000 children should represent the different districts of the city.

In order to satisfy these two conditions at least one district was selected from each of the ten groups * into which the city has been divided for testing purposes. The tests were given in twelve districts, and the following tables show the number of buildings, class rooms and grade classes tested, also the number of children in each grade.

Test in Addition of Fractions, December, 1915.

TABLE 1.

Number of districts tested	12
Number of buildings tested	12
Number of class rooms tested	88
Number of grade classes	91

TABLE 2.

Grade.	Number of Grade Classes.	Number of Pupils.
VI	31	1,265
VII	32	1,243
VIII	28	1,130
	91	3,638

* See School Document No. 10, 1915, Boston Public Schools.

b. Construction of Tests.

In constructing the tests the department used all of the material furnished them by the Committee on Standards in Arithmetic. By referring to the explanation of the types on page 5, it will be seen that there are certain types which are similar. As pointed out in discussing the results of the experimental work carried on under the direction of the committee, certain errors were made in one type but not in others similar to it. For example, a child might fail in doing Type 7 where the common denominator was one of the denominators and after adding it was necessary to reduce the answer to the lowest terms, but have no trouble in doing Type 5 where it was necessary to find a common denominator in the same way but the answer was non-reducible.

This same child might have had difficulty in doing Type 3 because, although it was not necessary to find a common denominator because the fractions were similar, it was necessary to reduce the answer to the lowest terms. Thus it was shown that a child who could do examples similar to Type 7 would probably be able to do examples similar to Types 3 and 5. The contrary, however, was not true for it was shown that some children who had no trouble with Type 5 would have trouble with Types 3 and 7. On the other hand, owing to the fact that the least common denominator was found in a different way in Type 7 than it was in Type 3, a child might be able to do Type 3 and not Type 7. It was thus decided that examples similar to Type 7 should compose one of the tests. In this way, examples from a list representing Types 8, 10, 13 and 14, together with those from Type 7, were decided upon as five of the tests. It was later decided to include examples similar to Type 3, as a means of determining the zero ability of children to do addition of fractions. That is, it was assumed that any child who could not do Type 3 would not be able to add fractions at all and a child who was just able to do Type 3 and no more would be able to handle only the simplest form of addition of fractions.

For the purpose of increasing the number of examples in Test 5, examples were selected from Type 11 and included with those from Type 13. The least common denominator of these examples was found in the same way, but the result in one case was non-reducible. Likewise, since there were only a few examples available similar to Type 14, it was necessary to select examples from Type 12. The least common denominator of each of these types was found in the same way, but in Type 12 it was only necessary to reduce the result to a mixed number.

The following, therefore, shows the type of example used in each test:

Test 1 consisted of examples similar to Type 3.
Test 2 consisted of examples similar to Type 7.
Test 3 consisted of examples similar to Type 8.
Test 4 consisted of examples similar to Type 10.
Test 5 consisted of examples similar to Types 11 and 13.
Test 6 consisted of examples similar to Types 12 and 14.

A folder containing the six tests was printed on sheets 8 by $10\frac{1}{2}$ inches with the following directions printed on the outside page:

This folder contains six tests in addition of fractions. You will be given two minutes in each test to add as many of the fraction examples as you can. Use no other paper. You are not expected to do them all. You will be marked for both speed and accuracy, but it is more important to have your answers right than to try a great many examples.

Each test consisted of twenty-four examples. This large number of examples was given not with the idea that a child of any grade should be able to complete the entire test in the time allowed, but that there should be enough examples to keep most of the children busy during the time allotment.

The following table shows four examples selected

from each test. Space was left at the right and below each example for the child to do any figuring which he deemed necessary.

TABLE 3.

Showing Examples Used in Tests in Addition of Fractions, December, 1915.

Addition of Fractions.— Test 1.— Time, 2 Minutes.

(1)	$\frac{1}{4}$	(2)	$\frac{3}{14}$	(3)	$\frac{5}{16}$	(4)	$\frac{1}{10}$
	$\frac{1}{4}$		$\frac{1}{14}$		$\frac{7}{16}$		$\frac{7}{10}$

Addition of Fractions.— Test 2.— Time, 2 Minutes.

(1)	$\frac{1}{3}$	(2)	$\frac{2}{7}$	(3)	$\frac{2}{3}$	(4)	$\frac{1}{3}$
	$\frac{1}{6}$		$\frac{3}{14}$		$\frac{1}{12}$		$\frac{7}{15}$

Addition of Fractions.— Test 3.— Time, 2 Minutes.

(1)	$\frac{3}{5}$	(2)	$\frac{5}{6}$	(3)	$\frac{5}{7}$	(4)	$\frac{14}{15}$
	$\frac{11}{15}$		$\frac{1}{2}$		$\frac{11}{14}$		$\frac{2}{3}$

Addition of Fractions.— Test 4.— Time, 2 Minutes.

(1)	$\frac{1}{7}$	(2)	$\frac{7}{9}$	(3)	$\frac{3}{4}$	(4)	$\frac{4}{9}$
	$\frac{9}{10}$		$\frac{1}{4}$		$\frac{3}{7}$		$\frac{5}{8}$

c. *Time Allowance.*

When the department was preparing the tests, the question of how long a time should be allowed for each test became a vital one. As shown by the experiment of the committee, the time used in doing twelve examples varies greatly. The department only desired to allow enough time to show the ability of the pupils to add fractions. As a means of determining this point, six

sheets were prepared similar to those used in the test given later in the elementary schools. The examples on these sheets were given to a class of forty students in the Normal School, using a time allowance of two minutes for each test. This time allowance was based in part on the records of pupils in using the practice material. It was found that enough examples were completed to give a measure of the ability of the individual pupil, and hence this time was allowed for each test.

d. Giving the Tests and Correcting the Results.

Following the plan of giving the Courtis tests, a group of nineteen seniors from the Normal School were trained to give the tests in a uniform manner. The tests were given to 1,265 children in Grade VI., 1,243 children in Grade VII., and 1,130 children in Grade VIII., on December 9, 1915. In addition to the instructions printed on the outside page of the folder, the examiners were instructed that if they were asked by children or teachers in regard to reduction to lowest terms or mixed numbers, to say that they were to do as they were in the habit of doing.

After giving the tests the examiners collected the folders and brought them to the office of the department. All correcting and tabulating of the results was done under the direction of the department. This method of handling the results insures, among other things, a uniformity of correction as well as a uniformity of giving the test. Any question respecting the correction of the work which the examiner might wish to ask was answered by the person in charge. If this question arose again on another paper, it was answered in the same way as the previous question. Thus it was possible to correct the papers in a much more uniform manner than under any other conditions. In this correction the main questions to be decided were whether the pupil had followed instructions or not. The following general rules were given the examiners to govern them in the correction of the papers.

(*a.*) All results which were not reduced to lowest terms or to mixed numbers were called wrong.

(*b.*) The papers on which children multiplied or subtracted the fractions were counted as I. N. F. papers (Instructions Not Followed).

(*c.*) All other papers, regardless of how the child did the example, were scored as right or wrong.

(*d.*) The form of doing the work did not count against the child if his answer was correct.

Analysis of Results.

a. Achievement.

Table 4 shows the results for the entire number of children tested. In the first column is shown the grade, followed by a column showing the number of pupils tested in each grade. Under each test are given the speed medians and the accuracy medians for each test and grade. The table is to be interpreted as follows: In Grade VI., 1,265 pupils were tested. These pupils attained a speed median of 10.7 examples with an accuracy median of 79.6 per cent in Test 1. In Test 2 the speed median is less, falling to 7.7 examples and the accuracy median falling to 65.6 per cent. Thus, reading across the page on the first line will be found the two medians for Grade VI. The table shows the same facts for Grades VII. and VIII., respectively.

TABLE 4.

Summary Sheet — City Medians.

Addition of Fractions, December, 1915.

Grade.	Pupils Tested.	Test 1.		Test 2.		Test 3.		Test 4.		Test 5.		Test 6.	
		Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.
VI.........	1,265	10.7	79.6	7.7	65.6	5.5	41.9	4.0	69.5	4.6	51.0	4.4	48.6
VII........	1,243	16.5	86.6	10.1	72.9	7.3	46.1	5.3	69.2	6.3	54.9	5.7	48.1
VIII.......	1,130	20.7	88.2	11.6	74.4	8.4	47.4	6.0	67.8	6.9	52.4	6.4	46.5

How far these figures may be relied upon as setting a standard to be attained in addition of fractions is doubtful. Many children paid no attention to reduction to mixed numbers or reduction to lowest terms; thus they did an abnormally large number of examples with a large number incorrect. This makes the speed median high and the accuracy median low. The only thing needed to make correct many of these incorrect results was the reduction to lowest terms. If this had been done as the child proceeded with his work, he would necessarily have done fewer examples with greater accuracy.

In the first test there were many cases of children who wrote answers to the entire twenty-four examples and reduced the results to lowest terms until the time expired. The result was a child might have had twenty-four examples attempted with five examples right. If instead of doing his work in this way he had reduced each example to lowest terms as he proceeded, his number attempted would have been less and his accuracy greater. This happened in so many cases in all of the tests that the standard medians in speed should probably be lower than those represented in the table and the standard accuracy higher. However, whether the results shown in Table 4 are higher or lower than a standard ought to be, they show what the department aimed to find out, namely, what the facts are concerning the ability of elementary school children to add fractions.

b. Kinds of Errors.

In constructing the tests it was intended to arrange them in the order of difficulty. If we judge whether this has been done or not upon the median speed with which a given grade performed the examples of a given test, Table 4 shows that apparently Test 4 is more difficult than either Test 5 or 6 because the number of examples done in the time allowed is slightly less in all three grades than was accomplished in the two following tests. The accuracy with which Test 4 was done, how-

ever, is slightly greater than in either of the two following tests. This result may be due in part to the fact that the examples represented by Type 4 are much more numerous than in either of the other types represented in the test. Therefore, chance would make it probable that the child would deal with this type of examples much oftener than with any of the remaining types. This would tend toward greater accuracy. The common denominator in this particular type may be larger and therefore might have a tendency to cause slower motor reactions, thus causing a slight reduction in the number of examples attempted. However, in view of the results indicated, it would seem that Tests 1 and 2 were easier than any of the others and that Tests 3, 4, 5, 6 are relatively of nearly the same difficulty. This is also borne out by the number of failures in these four tests.

In studying the results of Test 3 it will be noticed that although the speed median is only slightly smaller than in Test 2 and larger than in Test 4 or 5, the accuracy median is very low. This fact was so noticeable that it led to a detailed study of this test in order to discover the reason for this low percentage of accuracy. In order to obtain a correct answer in the examples in this test it was necessary for the children, after reducing to a common denominator and adding the numerators, to reduce the result thus obtained to both lowest terms and a mixed number. Under these conditions, the low accuracy was found to be due to three causes:

(*a.*) 19.1 per cent paid no attention to reducing to mixed numbers.

(*b.*) 17.6 per cent paid no attention to the reduction to lowest terms.

Nearly half of these paid no attention to either reduction.

(*c.*) Although they reduced to lowest terms and to a mixed number, 4.5 per cent expressed the answer in such a way that it did not tell the truth. For example, in adding $\frac{3}{5}$ and $\frac{11}{15}$ they obtained $\frac{20}{15} = 1\frac{5}{15} = \frac{1}{3}$ instead of $1\frac{5}{15} = 1\frac{1}{3}$.

In some cases where this was done, not only in Test 3 but in Test 6, it was questionable whether the children really understood that the answer was $1\frac{1}{3}$. Many of them did the example in the space at the right provided for that purpose and finally wrote their answer under the line beneath the example and wrote only the fraction, leaving out the whole number entirely. This would be of vital importance if mixed numbers were being added. Further, it seems that if we can tell the truth in our arithmetic work we should by all means do so. Probably no sign is more abused and misused than the sign of equality and if we could reinforce the meaning of the word "equals" in this particular case, we should take advantage of the opportunity to make the expression which is written tell the truth.

TABLE 5.

Per Cent of Pupils Obtaining Zero Examples Right in Fraction Test.

	VIII.	VII.	VI.
Number of pupils	1,130	1,243	1,265
Test 1	20.6	22.0	27.0
Test 2	35.0	24.4	34.2
Test 3	44.3	46.2	46.6
Test 4	43.3	28.6	31.7
Test 5	30.4	28.2	34.5
Test 6	42.9	33.8	39.8

One method of determining the standing of a class is to show the percentage of children who obtain a score of zero in the examples right. Table 5 shows the per cent of pupils in each grade who had no examples right in each separate test. That is, 20.6 per cent of the children in Grade VIII. had no examples right in the simplest test, Test 1; 22 per cent of the seventh grade and 27 per cent of the sixth grade did no better in the same test. The large per cent of the pupils obtaining a zero score in the third test is due, as pointed out in the foregoing, to the fact that they did not reduce either to

lowest terms or to mixed numbers. The failure to perform the necessary reductions is the cause of the high percentage of zero scores in all grades and in all tests. The relatively low per cent in Test 5 is probably due to the construction of the test. As previously indicated, the test consists of examples from Types 11 and 13. The inclusion of Type 11 was found to be a help to the children because many who were unable to obtain a correct answer to the first example which was taken from Type 13, because they did not reduce to the lowest terms, were able to obtain a correct answer to the second example which was taken from Type 11 and therefore did not have to be reduced.

The large per cent of zero scores in Test 6 may be due partially to the method of finding the least common denominator. The question of factoring has not been emphasized very largely in the last few years, and may raise a question as to whether such examples should have been included in the tests. In making up the test, however, fractions with large denominators were carefully avoided. In fact, in no case was the denominator of either fraction larger than 12 and the least common denominator in no case exceeded 60. It is the opinion of the department that such fractions are large enough for ordinary purposes, but that in no case should the denominator of the fractions to be added exceed 16 and that the least common denominator should be less than 100. This should make it possible for most children to determine the least common denominator by inspection and make it unnecessary to teach the forms of finding the least common denominator by factoring as taught in the various arithmetics.

c. *Proportion of Failures.*

There are two classes of children who obtain a score of zero in their work. To the first class belong the children who get zero because of inaccurate work. Their method of doing the example is correct. The second class include those children who get zero because they do not know the method to be followed.

In undertaking to find how many made a failure in adding fractions, only the second class were taken into consideration. That is, it was counted a failure on the part of the child only when he showed by his work that he had no conception as to the meaning of addition of fractions. The number of examples which he performed was generally small and the work accompanying those examples showed that he had little or no idea of reduction to a common denominator. In many cases he undertook to add the fractions without finding a common denominator at all. These failures fell under three heads.

(*a*.) Adding of numerators without considering a common denominator. In order to make the answer a fraction, the sums may have been placed over the sum of the denominators, the product of the denominators, or over either one of the denominators.

(*b*.) Adding all the figures as integers and either placing the sum over some denominator or allowing it to stand as an integer.

(*c*.) Apparently impossible ways of obtaining an answer.

Table 6 shows there were 70 children or 6.2 per cent in Grade VIII., 144 children or 11.6 per cent in Grade VII. and 108 children or 8.5 per cent in Grade VI., who made such failures in Test 1. In Test 2, 182 or 16.1 per cent in Grade VIII., 218 or 17.5 per cent in Grade VII., and 211 or 16.7 per cent in Grade VI. failed, and so on.

TABLE 6.

Number of Pupils and Per Cent Making Absolute Failures in Each Test in Addition of Fractions.

	VIII.		VII.		VI.	
	Pupils.	Per Cent.	Pupils.	Per Cent.	Pupils.	Per Cent.
Test 1	70	6.2	144	11.6	108	8.5
Test 2	182	16.1	218	17.5	211	16.7
Test 3	171	15.1	249	20.0	237	18.7
Test 4	196	17.4	261	20.9	279	22.1
Test 5	193	17.1	256	20.6	253	20.0
Test 6	193	17.1	256	20.6	240	18.9

Notice that about 17 per cent in Grade VIII., 20 per cent in Grade VII. and 20 per cent in Grade VI. failed to do Tests 3, 4, 5 and 6. This bears out what has been already stated, namely, that these four tests are apparently of nearly the same difficulty.

TABLE 7.

Number of Pupils and Per Cent Making Absolute Failures in Combined Tests in Addition of Fractions.

	VIII.		VII.		VI.	
	Pupils.	Per Cent.	Pupils.	Per Cent.	Pupils.	Per Cent.
Tests 1-2-3-4-5-6	57	5.0	144	11.6	104	8.2
Tests 2-3-4-5-6	96	8.5	71	5.7	97	7.7
Tests 3-4-5-6	7	.6	28	2.3	16	1.3
Tests 4-5-6	20	1.8	12	1.0	28	2.2
Tests 5-6	7	.6	1	.1	1	.1
Test 6	6	.5	0	.0	2	.2

Table 7 shows the number of pupils and the per cent making absolute failures in the combined tests in addition of fractions. That is, 57 pupils or 5 per cent of Grade VIII., 144 pupils or 11.6 per cent of Grade VII., 104 pupils or 8.2 per cent of Grade VI. failed in solving a single example in any test. If, as pointed out earlier in this report, Test 1 shows the zero ability of pupils to do addition of fractions, then from 5 per cent to 11 per cent possess this zero ability. Also 96 pupils or 8.5 per cent of Grade VIII., 71 pupils or 5.7 per cent of Grade VII., 97 pupils or 7.7 per cent of Grade VI. were unable to do anything beyond the first test. That is, from 5 per cent to 8 per cent are just able to do a simple example in addition of fractions when there are two addends. The table shows that, on the whole, if a child could do the first two tests, he could do the following four tests. Only a very small per cent failed in these last tests.

d. Causes of Errors.

In general there are three causes of inaccuracy.

(*a.*) Fundamental faults; that is, the children have

little or no idea how to find the least common denominator, how to reduce the fractions to this least common denominator, how to reduce to lowest terms or to mixed numbers. Many such children make a proper fraction equal to a whole or mixed number: for example, $\frac{3}{8} = 2\frac{2}{8}$.

(*b.*) Faults that are easily corrected. For example, the reduction to lowest terms and to mixed numbers in most cases requires only an explanation by the teacher, and then an insistence that all papers passed in shall have the result in its lowest terms or shall be reduced to a whole or mixed number.

(*c.*) Inaccuracy in the work which the child does. These are the same inaccuracies that occur in the manipulation of whole numbers and in all cases tend toward low scores in speed or in accuracy.

e. Methods Used in Doing the Examples.

Some of the methods used in doing the examples were ineffective. In some cases they caused the pupil to waste the time which he had at his disposal; in other cases the methods trained him to think in a wasteful manner or not to think at all.

Approximately one third found it necessary to reduce the fractions to a common denominator in the first test when the fractions were already similar. Some of these children wrote the fractions over a common denominator, using for a common denominator the denominator of the similar fractions. Others, not noticing that the fractions already had a common denominator, used some multiple, making the necessary reductions. For example, many children added $\frac{3}{14}$ and $\frac{1}{14}$ by reducing the fractions to a common denominator of 196. In many cases they then made errors in their work, thus obtaining an incorrect answer to the example. Even if carried through correctly, this is an ineffective and wasteful way of doing such examples.

Another method used by many individuals consisted of finding the least common denominator of such

fractions as $\frac{1}{8}$ and $\frac{3}{16}$ by finding the least common multiple of the denominators by short division as taught in many of the arithmetics. In such cases the following was found:

$$\begin{array}{r|l} 2 & 8 - 16 \\ \hline 2 & 4 - \ \ 8 \\ \hline 2 & 2 - \ \ 4 \\ \hline 2 & 1 - \ \ 2 \\ \hline & 1 - \ \ 1 \end{array} \qquad\qquad 2 \times 2 \times 2 \times 2 = 16$$

This indicates that the child is doing no thinking. The process is a wholly mechanical one and is not developing in the child the habit of correct or independent work. Such a method gives the child no basis for judging whether the answer is reasonable or not. No child who is thus tied to the mechanics of the operation has any opportunity for thinking about the result he is after. Oral work should take care of finding the common denominator in all cases of fractions with such small denominators.

f. Effect of Practice.

The tests in addition of fractions were given to three of the schools in which the material prepared by the Committee on Standards in Arithmetic was used. For purposes of comparison the department selected the grade in which the children who had used the practice material were enrolled in their respective schools. The papers of the selected grades in the three schools (termed in the following table, schools A, B, C) were then sorted and the results tabulated separately. Following this, the papers of the children who used the practice material (termed in the table the "selected group") were then sorted and the median scores computed.

TABLE 8.

Showing Comparison between Median Scores City Wide and Median Scores of Three Schools Using Practice Material.

Group.	Test 1.		Test 2.		Test 3.		Test 4.		Test 5.		Test 6.	
	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.	Speed Median.	Accuracy Median.
City	10.7	79.6	7.7	65.6	5.5	41.9	4.0	69.5	4.6	51.0	4.4	48.6
School A	12.9	48.8	9.0	43.6	6.7	29.7	5.6	34.2	6.0	36.6	6.2	32.0
School A, selected group	15.0	87.5	8.0	84.3	5.6	32.6	5.0	35.7	5.6	55.0	5.3	37.5
School B	8.3	88.3	7.4	70.0	5.1	45.7	3.8	84.4	4.7	60.9	4.0	61.4
School B, selected group	8.5	85.0	7.3	75.0	5.2	37.5	4.2	85.0	4.6	41.7	4.5	41.7
School C	12.7	46.2	9.4	41.2	7.2	38.3	4.6	64.6	5.7	46.9	5.3	50.0
School C, selected group	15.0	96.7	9.8	96.7	8.0	85.0	4.3	100.0	5.6	81.7	5.6	70.0

Table 8 shows the median scores of these three groups of children. It is to be interpreted as follows: In the line following the word "city" is shown the speed median and accuracy median for each test for Grade VI. in all the schools tested, as already shown in Table 4. In the next line the table shows that school A had a speed median of 12.9 examples with an accuracy median of 48.8 per cent in Test 1, and a speed median of 9 examples with an accuracy median of 43.6 per cent in Test 2, and so on. The selected group of school A has a speed median of 15 examples with an accuracy median of 87.5 per cent in Test 1, and so on.

The selected group in school A shows an advantage in speed in each test over the city-wide results. There is not always a superiority in this group over the results of the grade distribution of its own school. In spite of obtaining a lower median in speed, however, the group does surpass its own school in each test in accuracy. It fails to pass the city-wide accuracy median in Tests 3, 4 and 6. The selected group of school C surpasses the city-wide results in speed in each test and its own school in all but Tests 4 and 5. This group also surpasses both city-wide results and the results of its own school in accuracy.

The median for school B was comparatively high and the superiority of the selected group does not show so strikingly. It is plain, however, that even the small amount of individual training which was given during the spring, and may have been continued during the fall, has produced favorable results.

SUMMARY AND CONCLUSIONS.

1. The factors that enter into the problem of adding fractions are much more complex than those that enter into the problem of adding integers.

2. The errors were largely due to failing of pupils to reduce consistently either to lowest terms or to mixed numbers.

This failing on the part of many children to use the principle of reduction would seem to indicate that the method, now largely in use, of teaching such reductions by themselves, has failed to produce satisfactory results. In view of this fact, would it not be well to teach reductions, as such, in connection with the subject of addition of fractions? This would at least make a closer connection between the two operations, and thereby tend to form the habit of writing the answer in its best form.

3. Eight per cent in Grade VI., 11 per cent in Grade VII. and 5 per cent in Grade VIII. were unable to do the simplest problems in the addition of fractions.

4. Drilling and individual work given children in the schools in the spring showed its effect in the late fall. This was evidenced by an increase in both speed and accuracy over that obtained in the entire city and in two cases over that shown by the whole number of pupils in the grade in which the selected groups were enrolled.

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